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On the Perturbations of Planets and Comets. By Sir J. W. Lubbock, Bart.

The methods which form the basis of our tables of the older planets may be considered perfectly sufficient for that purpose, the sensible errors of those tables being due to the imperfect numerical developement of the theory, and not to any defect of the theory itself. These methods, however, suppose the orbits to be nearly circular and little inclined to each other, and are therefore inapplicable to the case of the periodical comets, and to that of several of the small planets. The only memoir, so far as the author is aware, which professes to gives a general solution of this problem, is due to M. Hansen, and is translated in the *Conn. des Temps* for 1847. In this memoir, M. Hansen has more particularly considered the case where the disturbed body is inferior, and has illustrated his method by the numerical calculation of the perturbations of Encke's comet by *Saturn*. The disturbing function is expressed in terms of sines and cosines of multiples of the excentric anomaly of the disturbed planet, and of the true anomaly of the disturbing planet. Advantage is then taken of the small excentricity of the orbit of the latter body, to convert the sines and cosines of multiples of the *true* anomaly into sines and cosines of multiples of the *mean* anomaly. The process, however, is very laborious, even in the case which M. Hansen has considered, and becomes far more so when *Jupiter* is the disturbing body. As the functions which require to be integrated are exhibited in terms of two variables, M. Hansen is obliged to integrate *by parts*, so that each term gives rise to a complicated series of other terms.

The method proposed by the author differs widely from that of M. Hansen. The numerical values of the elliptic constants are inserted at the earliest possible stage, no *literal* developement being attempted. When the disturbed body is inferior, the co-ordinates of the disturbing body may be expressed in terms of the excentric anomaly of the disturbed body, and thus the latter becomes the only variable involved in the resulting expressions, and the required integrations may be directly performed.

When the planet disturbed is superior to the disturbing planet, the only method which the author can suggest is to develope in terms of the true anomaly of the disturbed planet and the mean anomaly of the disturbing planet, and to integrate *by parts.*

Although a certain facility is afforded by the method of developement formerly proposed by the author (see *Monthly Notice*, vol. vii. No. 14), yet as the operations remaining are very complicated and liable to error, and as the disturbing function itself is never the *only* one whose developement is required, he now thinks it preferable to determine the coefficients of the developement by means of a sufficient number of particular values of the function to be developed. By this method any function, however complicated, may

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be developed as a whole; and the operations required, though long, are very simple.

The author has endeavoured to facilitate, as much as possible, the determination of the perturbations by the use of *Tables.* This can only be done in the case of a numerical developement.

In the present paper, the author considers only the case in which the orbit of the disturbed body is within that of the disturbing body.

The rectangular co-ordinates of any heavenly body may be very simply expressed in terms of its excentric anomaly. Tables have been calculated by Mr. Farley, and are given, containing the values of the constants which enter into these formulæ for the planets and periodical comets whose orbits are known.

Hence it is easy to obtain particular values of the co-ordinates and other quantities which are required in calculating the disturbing forces for given values of the excentric anomaly. Tables are given containing the values of these quantities for *Jupiter* and *Saturn*, corresponding to every fifth degree of excentric anomaly. These will be useful in every case of perturbations produced by those planets.

The disturbing force is usually decomposed into one of these two rectangular systems, viz. either a system of forces parallel to the axes of co-ordinates, or one in which two of the forces are in the plane of the orbit of the disturbed body, and respectively parallel and perpendicular to the radius vector, and the third is perpendicular to the plane of the orbit. The forces of the former system are given immediately in terms of the co-ordinates of the bodies, and those of the second system may be easily derived from them.

The well-known formulæ which give the differential coefficients of the elliptic elements with respect to the time are transformed into others, in which the excentric anomaly of the disturbed body is the independent variable. In these expressions the disturbing forces are multiplied by certain functions of the excentric anomaly, the values of which, corresponding to both of the above-mentioned systems, are given, and may be tabulated for given values of the excentric anomaly.

Hence may be found any number of particular values of the differential coefficients of the elliptic elements corresponding to given values of the excentric anomalies; and it only remains to obtain from these particular values the general developement of these quantities in such a form as to admit of being easily integrated.

The following is the method which the author employs for this purpose:—If ξ ξ' denote the mean anomalies, v v' the excentric anomalies, and n n' the mean motions, we may put

$$\xi' = \frac{n'}{n}\,\xi + \alpha, \quad \alpha \text{ being a constant}$$

$$= \frac{n'}{n}\,v + \alpha - \frac{n'}{n}\,e \sin v.$$

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Hence, if $\frac{n'}{n} v + \alpha$ be called $\mathfrak{n}$, the mean anomaly of the disturbing planet will never differ widely from $\mathfrak{n}$, since $\frac{n'}{n}$ is a fraction.

The author shews how to obtain the developement of the differential coefficients in terms of sines and cosines of multiples of v and $\mathfrak{n}$; and then, since $\mathfrak{n} = \frac{n'}{n} v + \alpha$, the required integrations may be immediately performed.

In conclusion, the author points out the relative advantages of the method of mechanical quadratures, and of those in which the values of the elliptic elements or the co-ordinates are expressed in series.

The Meeting then proceeded to the election of the Council for the ensuing year, when the following Fellows were elected, viz.:

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